quivering quicksand - Principles of Construction

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1 Introduction

The piece is based on research I did as a theoretical physicist in first half of the 1990s. At that time I was interested in a phenomenon, called Self-Organized Criticality. I studied it in some models. On of the models was the Frenkel-Kontorova model. This is a simple mechanical model of particles in a row connected by springs. The particles are in a spatially periodic force field. The particle motion is damped by a force proportional to their velocity. The space is one-dimensional. In my studied I let one end of the chain of particles moving very slowly.[1] This movement lead to instabilities which causes rearrangement of the particles. These rearrangement are like outbursts or 'avalanches' in the system.

The main idea for *quivering quicksand* was to map the positions of the particles onto the frequencies of some oscillators. In addition the kinetic energy of the particles should be used to control the amplitude of the oscillators.

Therefore the sound of *quivering quicksand* is created in four steps:

1. Integrate the Frenkel-Kontorova model numerically.
2. Map the time, positions and velocities onto the time, frequencies and amplitudes of the oscillators.
3. Choose the mapping parameters in accordance to a schema and specific events.
4. Pan the oscillator signals onto sound channels.

The piece should be presented in a large room with eight loudspeakers arranged in a circle. In addition there should be one or two subwoofers.

There are two different events which trigger a change in mapping parameters of step 3. One is triggered when the kinetic energy (which is roughly the overall loudness) is below a threshold. The second event happens when a person is close to a ultrasonic distance detector which should be installed near the entrance of the room of the sound installation.

[1] Franz-Josef Elmer: Self-Organized Criticality in the Weakly Driven Frenkel-Kontorova Model, Helvetica Physics Acta, Vol. 66, 1993

2 The weakly driven Frenkel-Kontorova Model

The Frenkel-Kontorova model is defined by the following set of differential equations:

$$\ddot{x}_i + g\dot{x}_i + \partial V/\partial x_i = 0, \quad i = 1, ..., N, \tag{1}$$

where V (the external potential, causing the spatially periodic force field) is defined by

$$V = \sum_{i=1}^{N} \frac{1}{2}(x_i - x_{i-1} + a)^2 - b\cos x_i \tag{2}$$

x_i is the position of the ith particle, g is the damping constant, a the equilibrium length of the springs, and b is the strength of the external potential.

There are $N+1$ particles, $x_0, ..., x_N$. The particles x_i, for $i > 0$ are governed by the differential equations specified above. But the particle x_0 is controlled externally: It will be moved by small constant velocity v, i.e. $x_0(t) = x_0(0) + v \cdot t$.

The set of differential equations will be integrated numerically with the fixed step size Δt. For more details see Appendix A.

3 Mapping

The sound of *quivering quicksand* is created by N sinusoidal oscillators.

3.1 Frequency Mapping

The frequencies f_i of these oscillators are determined by the positions x_i by a linear mapping:

$$f_i = f_{\text{offset}} + f_{\text{scale}} \cdot (x_i - x_{index}) \tag{3}$$

3.2 Amplitude Mapping

The amplitudes A_i of the oscillators are determined by the velocities $\dot{x}_i$ or more precisely by the kinetic energy $\dot{x}_i^2/2$. The mapping is not a simple linear mapping of the kinetic energy to the amplitude. First, the kinetic energy is discretized. Second, the discretized kinetic energy is smoothed over time. Third, from the smoothed discretized kinetic energy the amplitude is calculated.

Discretizing

The kinetic energy

$$E_i = \frac{\dot{x}_i^2}{2} \tag{4}$$

is discretized by the discretization parameter h as follows:

$$E_i^d = \text{floor}\left(\frac{E_i}{\tilde{h}}\right)\tilde{h}, \quad \text{with} \quad \tilde{h} = \frac{h}{N}\sum_{i=1}^{N} E_i, \tag{5}$$

where floor(x) is the largest integer $\leq x$.

This discretization leads to the musical effect that for $h \gg 1$ almost all E_i^d are zero except for a few. E_i^d will be scaled up by a factor in order to avoid that the sound becomes too low compared to cases where $h \ll 1$. The rescaled discretized kinetic energy reads

$$\tilde{E}_i^d = \left(\frac{\sum_{i=1}^{N} E_i}{\sum_{i=1}^{N} E_i^d}\right)^2 E_i^d \tag{6}$$

Smoothing

In order to smooth $\tilde{E}_i^d(t)$ the following linear differential equation is solved:

$$\frac{d\bar{E}_i^d}{dt} = -\frac{\kappa}{\Delta t} \cdot \left(\bar{E}_i^d - \tilde{E}_i^d(t)\right) \tag{7}$$

The smoothed discretized kinetic energy at time point $t_{k+1} = (k+1) \cdot \Delta t$ reads:

$$\bar{E}_i^d(t_{k+1}) = \left(\bar{E}_i^d(t_k) - \tilde{E}_i^d(t_k)\right)e^{-\kappa} + \tilde{E}_i^d(t_{k+1}) - \left(\tilde{E}_i^d(t_{k+1}) - \tilde{E}_i^d(t_k)\right)\frac{1 - e^{-\kappa}}{\kappa} \tag{8}$$

For more details see Appendix B.

Amplitude

The amplitude A_i of oscillator i at time point t_k is calculated as follows:

$$A_i(t_k) = A_0\sqrt{\bar{E}_i^d(t_k)}. \tag{9}$$

3.3 Tempo Mapping

For determine the tempo the total kinetic energy

$$E_{total} = \frac{1}{N}\sum_{i=1}^{N}\frac{\dot{x}_i^2}{2} \tag{10}$$

is smoothed similarly as the discretized kinetic energy for the amplitude mapping. The smoothed total kinetic energy $\bar{E}_{total}$ is calculated in accordance with Appendix B by using $\mu = g$.

The tempo is determined by the rate of calculating the solution of the Frenkel-Kontorova model for the next time point. The real time step ΔT is determined by

$$\Delta T = \text{floor}(\rho A_0 \sqrt{\bar{E}_{total}}) \tag{11}$$

4 Distribution of Oscillator Signals onto Channels

It is assumed that there are M loudspeakers (except of the subwoofers) arranged in a circle. For each oscillator i a virtual position on the circle is determined by

$$s_i = \eta \cdot \left(\frac{x_i}{aN} + 1 \right) \tag{12}$$

The signal is distributed over all M channels where the channels near the virtual position get the largest amplitudes. This distribution is determined by the following function

$$\beta(s) = \text{floor}(s) + \frac{1}{2} + \frac{2\bar{s} - 1}{2\sqrt{\frac{2\bar{s}(1-\bar{s})}{\sigma} + (2\bar{s} - 1)^2}}, \quad \text{with} \quad \bar{s} = \text{mod}(s, 1) \tag{13}$$

This is a function which has steps (of size σ) at integer values. The value between the steps is given by floor$(s) + 1/2$.

The amplitude A_i^m of oscillator i for channel m is determined by

$$A_i^m = A_i \sqrt{\beta(\xi_i^m + \Delta\xi_m) - \beta(\xi_i^m - \Delta\xi_m)} \tag{14}$$

where A_i is determined by Eq. (9) and

$$\xi_i^m = \frac{m-1}{M} - s_i \quad \text{and} \quad \Delta\xi_m = \frac{1}{2M} \tag{15}$$

Note, that $\sum_m |A_i^m|^2 = A_i^2$ because $\beta(s+1) = \beta(s) + 1$.

5 Parameter Combinations

There are many parameters which determine the sound. Their actual values are kept constant until an event happens and new values are chosen. Infinite many values are possible because most of the parameters can have values from a continuum of numbers.

This has been restricted to finite sets of values for a subset of parameters p_j, for $j = 1, ..., J$:

$$p_j \in \{p_{j1}, p_{j2}, ..., p_{jl_j}, ..., p_{jL_j}\}, \quad \text{for} \quad j = 1, ..., J \tag{16}$$

At any time a given combination $l_1, l_2, ..., l_J$ is chosen. The total number of combinations is

$$N_C = \prod_{j=1}^{J} L_j \tag{17}$$

The sequence of combinations is chosen in accordance to the following rules:

1. The next combination differs from the previous one in as many parameters as possible in order to have a large contrast.
2. All combinations appear in a fixed cyclic order. That is, after the last combination the first appears again.

See Appendix C for the algorithm which calculates the next combination following these rules.

6 Events

The next parameter combination is chosen by two different events.

6.1 Kinetic Energy Event

For this event the smoothed total kinetic energy $\bar{E}_{total}$ (as introduced in subsection 3.3) is used. The event is triggered when

$$A_0 \sqrt{\bar{E}_{total}} < \underline{\theta}_K \tag{18}$$

with threshold $\underline{\theta}_K$. In order to prevent an immediate next event the trigger is inactive as long as

$$A_0 \sqrt{\bar{E}_{total}} < \bar{\theta}_K \tag{19}$$

with $\underline{\theta}_K < \bar{\theta}_K$.

6.2 Distance Event

An ultrasonic distance sensor[2] measures continuously the distance to persons in its vicinity. The event is triggered when the distance is below the threshold θ_D. The trigger is inactive during the recovery time $T_{recover}$.

[2]Arduino Uno WiFi Rev2 microcontroller with HC-SR04 ultrasonic sensor

7 Chosen Parameters

The actual audio experience of *quivering quicksand* depends strongly on the chosen parameters.

Here is a list of all chosen parameters, grouped as they have been introduced in the sections above:

Frenkel-Kontorova Model (section 2)

$$N = 34,\ a = 0.5,\ b = 3,\ g = 0.01,\ v = 0.01,\ \Delta t = 0.1$$

Frequency Mapping (subsection 3.1)

$$\begin{aligned} f_{\text{offset}} &= 300 \\ f_{\text{scale}} &\in \{1, 3, 8, 21\ \} \\ x_{index} &\in \{0, 34\} \end{aligned}$$

The frequencies f_{offset} and f_{scale} are in Hz.

Amplitude Mapping (subsection 3.2)

$$\begin{aligned} \kappa &\in \{0.01, 0.1, 1\} \\ h &\in \{0.5, 1, 2, 4, 8\} \\ A_0 &= 0.03 \end{aligned}$$

Tempo Mapping (subsection 3.3)

$$\rho = 200 \text{ msec}$$

Channel Distribution (section 4)

$$\eta = 0.1,\ \sigma = 0.02,\ M = 8$$

Kinetic Energy Event (subsection 6.1)

$$\underline{\theta}_K = 0.003,\ \bar{\theta}_K = 0.006$$

Distance Event (subsection 6.2)

$$\theta_D = 150 \text{ cm},\ T_{recover} = 2000 \text{ msec}$$

Note, that there are $120 = 4 \cdot 2 \cdot 3 \cdot 5$ different combinations.

A Verlet Algorithm

In order to solve the Ferkel-Kontorova model numerically the Verlet algorithm adapted for a damped system is used:

$$x_i(t_{n+1}) = x_i(t_n) + p_i(t_n)\Delta t + \big(f_i(x(t_n)) - gp_i(t_n)\big)(\Delta t)^2/2 \tag{20}$$

$$p_i(t_{n+1}) = \frac{p_i(t_n) + [f_i(x(t_n)) + f_i(x(t_{n+1})) - gp_i(t_n)]\Delta t/2}{1 + g\Delta t/2} \tag{21}$$

where $f_i(x) = -\partial V/\partial x_i$, $p_i = \dot{x}_i$, and $t_n = n\Delta t$.

B Smoothing A Signal

The smoothed signal $\bar{S}(t)$ is the solution of the differential equation

$$\dot{\bar{S}} = -\mu \cdot \big(\bar{S} - S(t)\big), \tag{22}$$

We assume that the signal $S(t)$ is a piecewise linear function of t where $S(t)$ is specified at $t_k = k \cdot \Delta t$ for $k = 0, 1, 2, \cdots$.

Eq. (22) is a linear inhomogeneous first order differential equation. The standard solving Ansatz is the solution of the homogeneous differential equation with a time dependent integration constant $C(t)$:

$$\bar{S}(t) = C(t)e^{-\mu(t-t_k)} \tag{23}$$

This leads to

$$C(t) = \bar{S}(t_k) + \mu \int_{t_k}^{t} S(t')e^{\mu(t'-t_k)}dt' = \bar{S}(t_k) + S(t')e^{\mu(t'-t_k)}\Big|_{t_k}^{t} - \int_{t_k}^{t} \dot{S}(t')e^{\mu(t'-t_k)}dt' \tag{24}$$

Thus, we get for $t = t_{k+1}$

$$\bar{S}(t_{k+1}) = \big(\bar{S}(t_k) - S(t_k)\big)e^{-\mu\Delta t} + S(t_{k+1}) - \big(S(t_{k+1}) - S(t_k)\big)\frac{1 - e^{-\mu\Delta t}}{\mu\Delta t} \tag{25}$$

C Choosing the Next Combination

The following algorithms fulfills the rules described in section 5 (not yet proven mathematically). The next value of the index l_j of parameter p_j is given by

$$\text{mod}(\text{floor}(\nu\alpha_j), L_j) \tag{26}$$

where $\nu \in \{0, 1, ..., N_C\}$ is the combination index and α_j a rational number which is calculated as follows:

$$\alpha_j = \frac{P_j}{Q_j} \tag{27}$$

where

$$Q_j = \begin{cases} 1, & \text{if } j = 1 \\ \prod_{i=1}^{j-1} L_i, & j > 1 \end{cases} \tag{28}$$

and P_j is the smallest number $\geq Q_j$ which has no common devisor with L_j. Note, that the next combination index $\nu_{next} = \text{mod}(\nu_{previous} + 1, N_C)$.